Energy Conversion

2nd year electrical

Chapter 3: Transformers

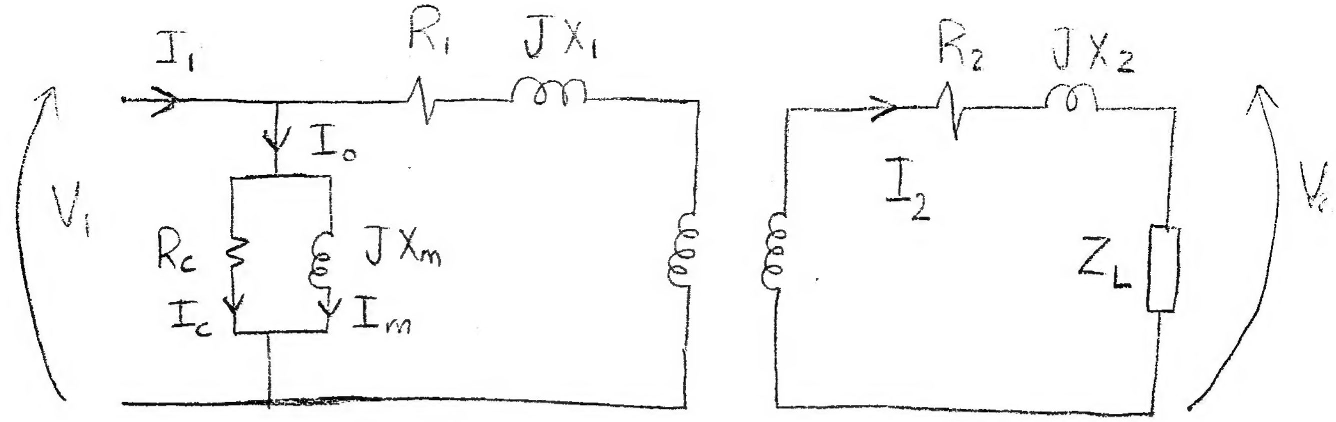
Part (2)

Paper:

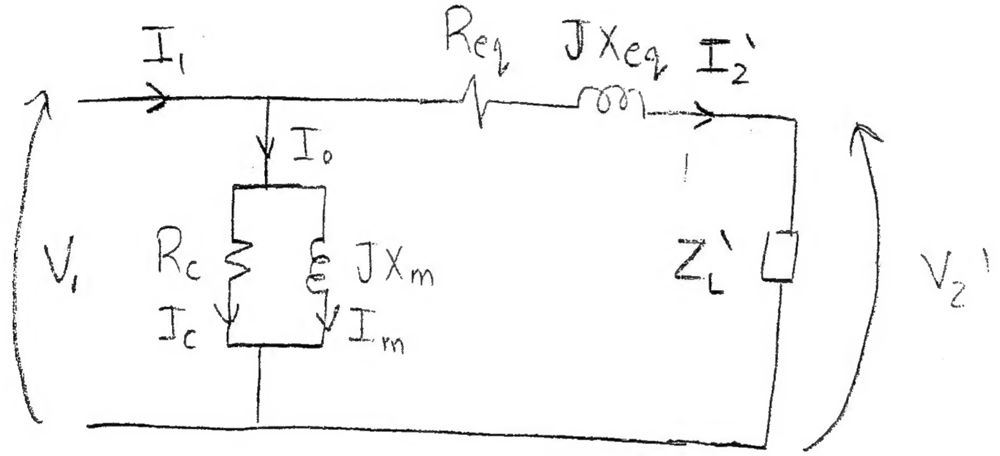
Transformers - Continued

1. Approximate equivalent circuit
2. Voltage regulation + phasor diagram
3. Efficiency (η)
4. Transformer tests → open circuit test
   → short circuit test
5. Separation of iron losses → $P_{hystersis}$
   → P_{eddy}

① Approximate equivalent circuit



→ Referring to primary side



* $R_{eq} = R_1 + R_2'$

  $R_{eq} = R_1 + a^2 R_2$

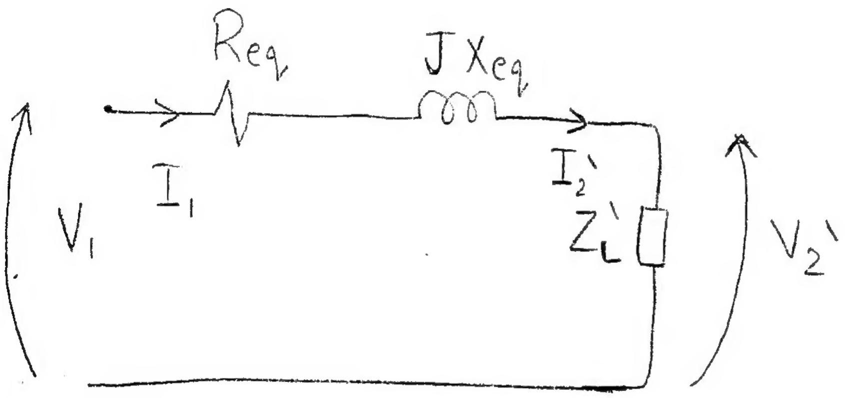
* $X_{eq} = X_1 + X_2'$

  $X_{eq} = X_1 + a^2 X_2$

(2) Transformer Voltage regulation (VR)

Definition: It's a measure of the change of voltage from no load to any loading condition. ($VR \leq 5\%$)

→ Consider the equivalent circuit of the transformer as follows:



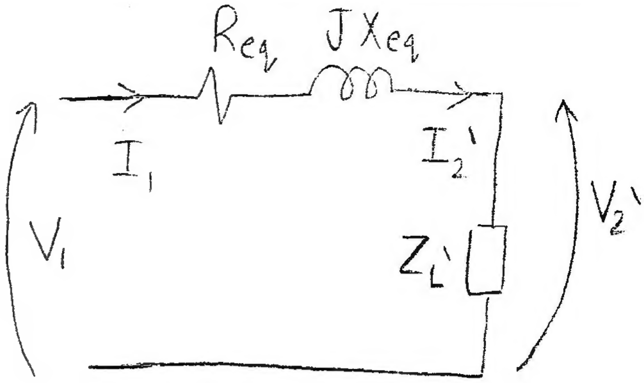
$$\boxed{\text{Voltage regulation} = \frac{V_1 - V_2'}{V_1}}$$

→ <u>at no-load</u> → $I_2' = 0$ → $V_1 = V'_{2\,no\text{-}load}$

$$\boxed{\therefore \text{Voltage regulation} = \frac{V_1 - V_2'}{V_1} = \frac{V'_{2\,no\text{-}load} - V_2'}{V'_{2\,no\text{-}load}}}$$

Voltage regulation for lagging p.f load

$$VR = \frac{V_1 - V_2'}{V_1}$$

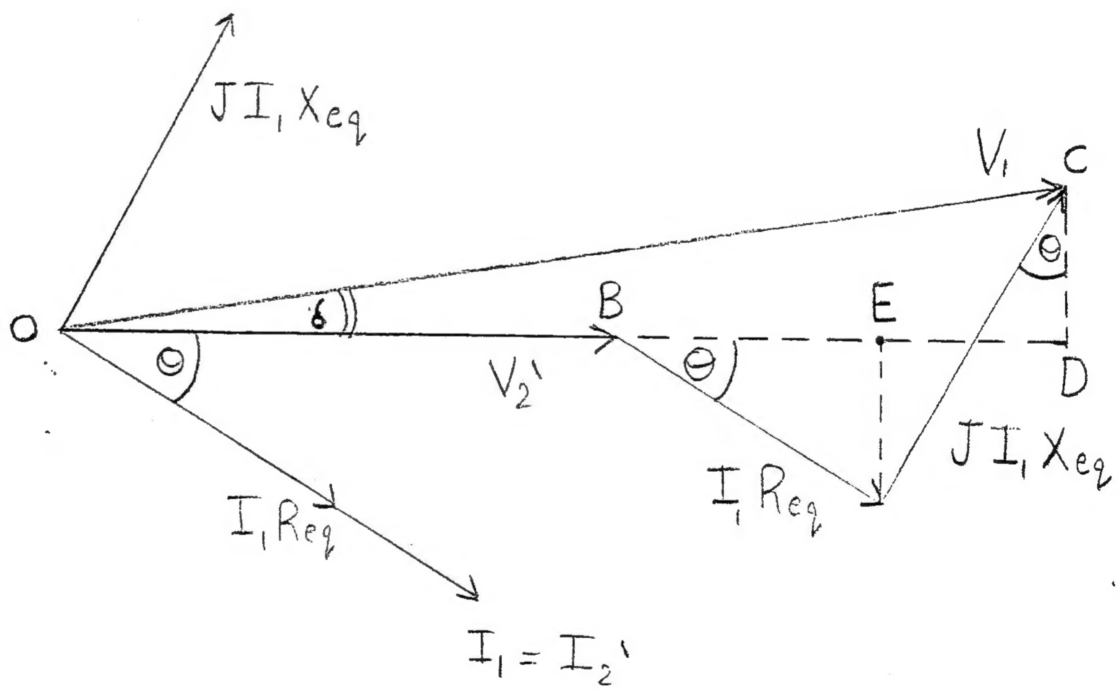


→ **KVL**

$$\overline{V_1} = \overline{V_2} + \overline{I_1}\,(R_{eq} + JX_{eq})$$

$$\overline{V_1} = \overline{V_2} + \overline{I_1}\,R_{eq} + J\overline{I_1}\,X_{eq}$$ → phasor equation

→ **phasor diagram**



To get $|V_1|$

$|V_1| = OC \simeq OD$ (as δ is very small angle)

$$|V_1| = OD = \underbrace{OB}_{|V_2'|} + \underbrace{BE}_{I_1 R_{eq}\cos\theta} + \underbrace{ED}_{I_1 X_{eq}\sin\theta}$$

$$= |V_2'| + I_1 R_{eq} \cos\theta + I_1 X_{eq} \sin\theta$$

$$\therefore |V_1| = |V_2'| + I_1 R_{eq} \cos\theta + I_1 X_{eq} \sin\theta$$

$$|V_1| - |V_2'| = I_1 R_{eq} \cos\theta + I_1 X_{eq} \sin\theta$$

$$\therefore VR = \frac{|V_1| - |V_2'|}{|V_1|}$$

$$\therefore VR = \frac{I_1 R_{eq} \cos\theta + I_1 X_{eq} \sin\theta}{V_1}$$ → For lagging PF

→ Condition for Maximum Voltage regulation

$$\frac{d\,(VR)}{d\theta} = 0 \implies \frac{d}{d\theta}\left[\frac{I_1 R_{eq} \cos\theta + I_1 X_{eq} \sin\theta}{V_1}\right] = 0$$

$$\therefore I_1 R_{eq}(-\sin\theta) + I_1 X_{eq} \cos\theta = 0$$

$$\therefore \tan\theta = \frac{X_{eq}}{R_{eq}}$$

$\therefore$ The load angle for maximum VR is

$$\theta_{max} = \tan^{-1} \frac{X_{eq}}{R_{eq}} \quad \text{(lagging pf)}$$

$$\therefore VR_{max} = \frac{I_1 R_{eq} \cos\theta_{max} + I_1 X_{eq} \sin\theta_{max}}{V_1}$$

③ Efficiency of transformers (η)

$$\eta = \frac{P_{o/p}}{P_{i/p}} = \frac{P_{o/p}}{P_{o/p} + P_{loss}}$$

where $P_{loss} = P_{cu} + P_{core}$

⟹ To get a general expression for η → هام

$$\eta = \frac{P_{o/p}}{P_{o/p} + P_{cu} + P_{core}}$$

* Let X = loading ratio = loading factor

$$X = \frac{I_2}{I_{2\,f.L}}$$

$$X = \frac{I_2 * V_2}{I_{2\,f.L} * V_2} = \frac{S}{S_{f.L}}$$

$$\therefore X = \frac{I_2}{I_{2\,f.L}} = \frac{S}{S_{f.L}}$$

⟹ ① $P_{o/p} = V_2 I_2 \cos\phi_L = V_2\, I_{2\,f.L} * \left(\frac{I_2}{I_{2\,f.L}}\right) \cos\phi_L$

$$P_{o/p} = X\, S_{f.L} \cos\phi_L \rightarrow ①$$

⟹ ② P_{core} = Constant → ②

$\Longrightarrow$ (3) $P_{cu} \propto I^2$

$$P_{cu} = X^2 P_{cu_{F.L}} \rightarrow (3)$$

From (1), (2), (3) we get:

حفظ

$$\eta = \frac{X S_{F.L} \cos\phi_L}{X S_{F.L} \cos\phi_L + X^2 P_{cu_{F.L}} + P_{core}}$$

To get η_{max}

$$\frac{d\eta}{dX} = 0 \Longrightarrow X\Big|_{\eta_{max}} = \sqrt{\frac{P_{core}}{P_{cu_{F.L}}}}$$

حفظ

$$\eta_{max} = \frac{\left(\sqrt{\frac{P_{core}}{P_{cu_{F.L}}}}\right) * S_{F.L} * \cos\phi_L}{\left(\sqrt{\frac{P_{core}}{P_{cu_{F.L}}}}\right) * S_{F.L} * \cos\phi_L + 2 P_{core}}$$

Note: Transformer efficiency is very high ($\eta_{trans} \simeq 95\%$) as it has no rotational losses

Ex: 1

The parameters of the equivalent circuit of a 10KVA, 50HZ, 2300/230V distribution transformer are:

$r_1 = 3.6\,\Omega$, $X_1 = 15.8\,\Omega$

$r_2 = 0.0396\,\Omega$, $X_2 = 0.158\,\Omega$

Where 1, 2 refers to HV, LV respectively

(a) If the transformer delivers its rated KVA at 0.8 pf lagging to a load on low-tension (LV) side, Calculate Voltage regulation.

(b) Calculate the efficiency in part (a) if the iron losses are 75W at rated Voltage.

Solution

(a) To get $I_{2\,rated}$ at LV side

$$|I_{2\,rated}| = \frac{S_{rated}}{V_2} = \frac{10\,000}{230} = 43.48\ A$$

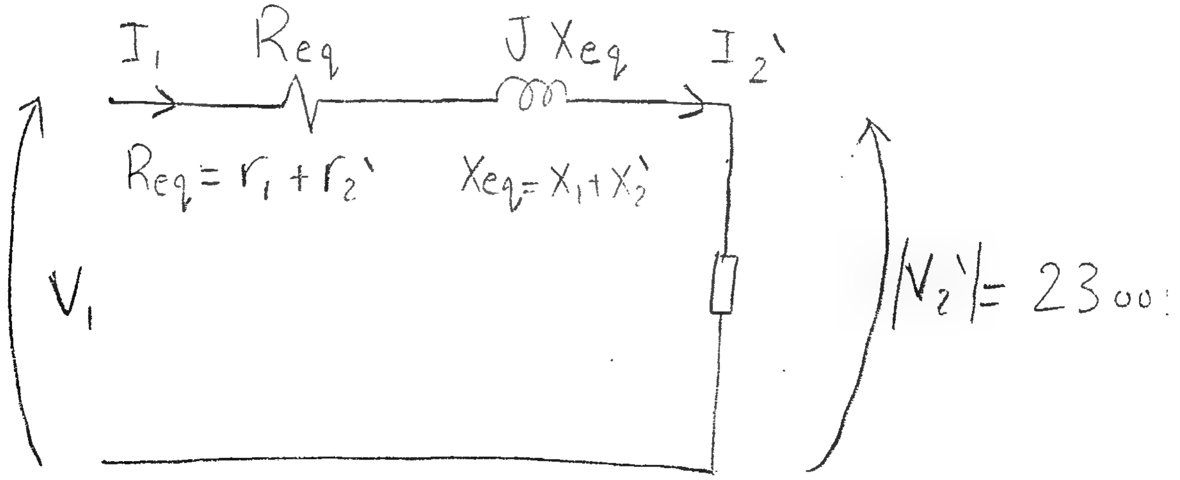
<u>Referring to HV side</u>

$$|V_2'| = a\,|V_2| = 10 * 230 = 2300\,V$$

$$|I_2'| = \frac{1}{a}\,|I_2| = \frac{1}{10} * 43.48 = 4.348\ A$$

$$r_2' = a^2\, r_2 = (10)^2 * 0.0396 = 3.96\,\Omega$$

$$X_2' = a^2\, X_2 = (10)^2 * 0.158 = 15.8\,\Omega$$



$$R_{eq} = r_1 + r_2' = 7.56\,\Omega$$

$$X_{eq} = X_1 + X_2' = 31.6\,\Omega$$

$$\phi = \cos^{-1} 0.8 = 36.87^\circ \text{ (lagging)}$$

Using laws of Voltage regulation : $(I_1 = I_2')$

$$V_1 - V_2' = I_1 R_{eq} \cos\phi + I_1 X_{eq} \sin\phi$$

$$V_1 = V_2' + I_2' R_{eq} \cos\phi + I_2' X_{eq} \sin\phi$$

$$\therefore V_1 = 2300 + (4.348)(7.56)\cos(36.87) + (4.348)(31.6)\sin(36.87)$$

$$V_1 = 2408.7 \text{ V}$$

$$\therefore VR = \frac{V_1 - V_2'}{V_1} = \frac{2408.7 - 2300}{2408.7}$$

$$VR = 4.5\%$$

(b) Efficiency

$$\eta = \frac{X S_{F.L} \cos\phi}{X S_{F.L} \cos\phi + X^2 P_{cu_{F.L}} + P_{core}}$$

* $X = 1$
* $S_{F.L} = 10\,000$ VA , $\cos\phi = 0.8$
* $P_{cu_{F.L}} = (I_2')^2 R_{eq} = (4.348)^2 * 7.56 = \underline{143 \text{ W}}$
* $P_{core} = P_{iron} = \underline{75 \text{ W}}$

$$\therefore \eta = 97.34\%$$

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pb (7 ii) Sheet 3

Givens: • $S_{F.L} = 500\,KVA$

• $\eta|_{X=1}$ (Full load) $= 0.95$

• $\eta|_{X=0.6} = 0.95$

• $p.f = 1$

(a) Separate out the transformer losses using the law of η

$$\eta = \frac{P_{o/p}}{P_{i/p}} = \frac{P_{o/p}}{P_{o/p} + P_{loss}} = \frac{P_{o/p}}{P_{o/p} + P_{cu} + P_{core}}$$

$$\eta = \frac{X S_{F.L} \cos\Phi_L}{X S_{F.L} \cos\Phi_L + X^2 P_{cu_{F.L}} + P_{core}}$$

$$\eta|_{X=1} = \eta|_{X=0.6} = 0.95$$

$$\Rightarrow \eta|_{x=1} = 0.95$$

$$\frac{1 * \overset{S_{F.L}}{500} * \overset{p.f}{1}}{1*500*1 + (1)^2 P_{cu_{F.L}} + P_{core}} = 0.95$$

$$\boxed{\therefore P_{cu_{F.L}} + P_{core} = 26.31 \text{ KW}} \rightarrow (1)$$

$$\Rightarrow \eta|_{x=0.6} = 0.95$$

$$\frac{0.6 * 500 * 1}{0.6*500*1 + (0.6)^2 * P_{cu_{F.L}} + P_{core}} = 0.95$$

$$\boxed{\therefore 0.36 P_{cu_{F.L}} + P_{core} = 15.79 \text{ KW}} \rightarrow (2)$$

Solving (1) & (2) $\Rightarrow$ $P_{cu_{F.L}} = 16.4$ KW, $P_{core} = 9.87$ KW

(b) $\eta|_{x=0.75}$, $p.f = 1$

$$\eta|_{x=0.75} = \frac{0.75 * 500 * 1}{0.75*500*1 + (0.75)^2 * \underbrace{16.4}_{P_{cu_{F.L}}} + \underbrace{9.87}_{P_{core}}}$$

$$\therefore \eta|_{x=0.75} = 95.15\%$$

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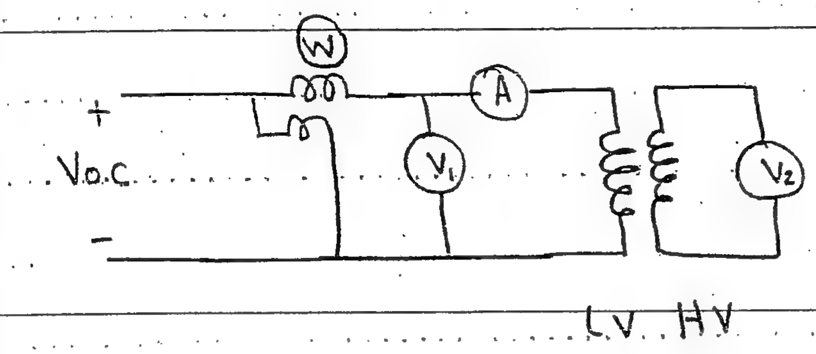
(IV) Transformer Tests:

(1) Open circuit Test:-

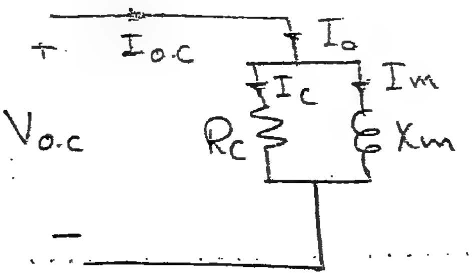
- This test is performed to find (R_c & X_m)



- $V_{o.c} \equiv$ Applied rated voltage $\equiv$ V_1
- $I_{o.c} \equiv$ open circuit current $\equiv$ A
- $P_{o.c} \equiv$ open circuit transformer power $\equiv$ W

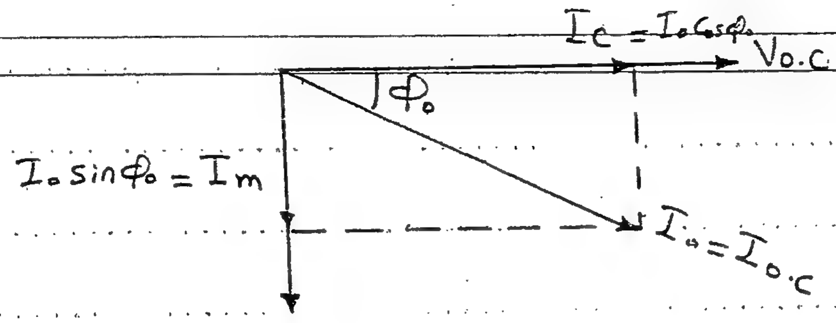
- How to get R_c & X_m?

- The equivalent circuit of the transformer during the open circuit test will be:



$$\therefore R_C = \frac{V_{o.c}}{I_C} \quad , \quad X_m = \frac{V_{o.c}}{I_m}$$

The values of I_C & I_m Can be Calculated through:



$\therefore$ $P_{o.c}$ will be used to get ϕ_o, then using the above phasor we draw $I_{o.c}$ lagging $V_{o.c}$ by ϕ_o, then resolve $I_{o.c}$ to (I_C & I_m).

$\therefore$ $P_{o.c} = V_{o.c}\, I_{o.c} \cos\phi_o \Rightarrow \phi_o$ Can be Calculated.

$$\boxed{\therefore \cos\phi_o = \frac{P_{o.c}}{(V_{o.c})(I_{o.c})}} \Rightarrow \phi_o = \text{o.k}$$

$$\therefore \boxed{I_C = I_o \cos\phi_o \ \& \ I_m = I_o \sin\phi_o}$$

$$\therefore \boxed{R_C = \frac{V_{o.c}}{I_{o.c}} \ \& \ X_m = \frac{V_{o.c}}{I_m}}$$

N.B :

[1] $I_m = \sqrt{I_{o.c}^2 - I_c^2}$ (Another way).

[2] R_c & X_m are referred to (L.V) side

[3] $I_{o.c}$ = no-load current

[4] $\cos\phi_o$ = no-load power factor.

[5] $V_{o.c} \equiv$ rated voltage of the transformer if not given.

[6] Instruments are connected to L.V side (why?)
(مرجع الـ نظري)

- Usually ($I_{o.c} = I_o$) is very small compared to the full load current. ($\simeq$ %5 $I_{full\ load}$)
- This small current is measured through the L.V of the transformer for better accuarcy.

N.B

- Usually the current in H.V side is lower than that of L.V

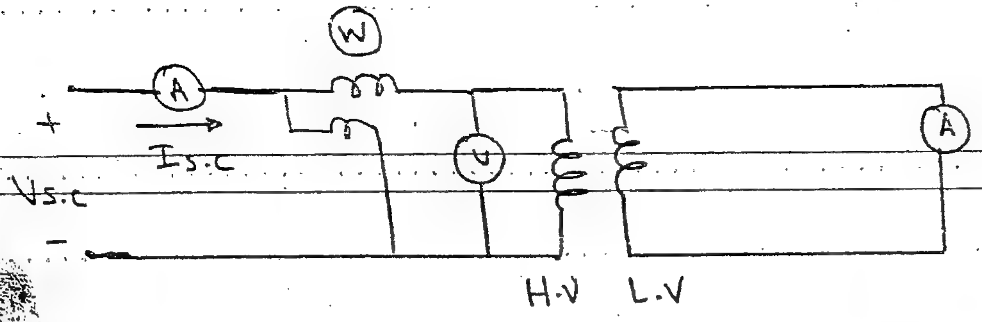
$$\because V_1 I_1 = I_2 V_2 \Rightarrow \frac{I_2}{I_1} = \frac{V_1}{V_2}.$$

$$\rightarrow \text{If } \frac{V_1}{V_2} > 1$$

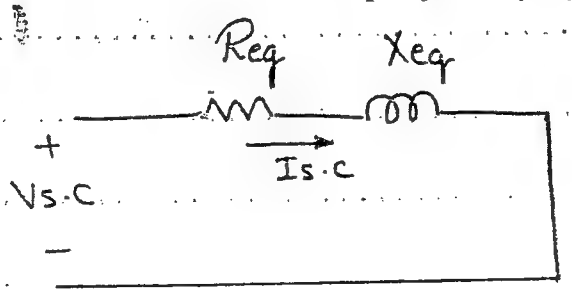
$$\therefore I_2 > I_1 \Longrightarrow \text{our case}...$$

(2) Short Circuit test:-

- It is performed to find Req & Xeq.



- $V_{s.c}$ = 5 – 10% of rated voltage of H.V side ≡ (V)
- $I_{s.c}$ = short circuit transformer Current ≡ (A)
- $P_{s.c}$ = short circuit transformer power ≡ (W)
- The equivalent circuit during this test:



- The (Rc & Xm) branch is neglected as the current (I_o) is very small with respect to $I_{s.c}$.
- To get (Req):

$$\boxed{R_{eq} = \frac{P_{s.c}}{I^2_{s.c}}}$$

To get X_{eq}:

$$Z_{eq} = \sqrt{R_{eq}^2 + X_{eq}^2} = \frac{V_{s.c}}{I_{sc}}$$

$$\boxed{\therefore \; X_{eq} = \sqrt{\left(\frac{V_{s.c}}{I_{s.c}}\right)^2 - R_{eq}^2}}$$

N.B.:

1. $I_{s.c} \cong I_{f.L}$ of transformer H.V side.

2. (R_{eq} & X_{eq}) are referred to the H.V side.

3. The instruments are placed on H.V side. (why?)

(مهمة برضه _ نظري)

∴ In this test the current flowing ($I_{s.c}$) is very high ($\cong I_{f.L}$)

∴ This current will be very high in the L.V side and it will very difficult to measure it by an ammeter.

. The voltage ($V_{s.c} \cong 5-10\% \; V_{rated}$), so it will be more accurate to measure it using the H.V side.

4 you Can use:

$$\boxed{r_1 = r_2' = \frac{R_{eq}}{2}}$$

&

$$\boxed{X_1 = X_2' = \frac{X_{eq}}{2}}$$

5 The open circuit test & short circuit test may be performed on any side, but it will be mentioned in the problem.

6 <u>Default:</u> (not mentioned in the problem).

- open Circuit test $\Rightarrow$ on L.V side
- short Circuit test $\Rightarrow$ on H.V side.

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7 Cross-section Area of LV Winding is large as it carries higher current
$\therefore R_{LV}$ is small $\left(R = \frac{\rho L}{A}\right)$

8 Cross-section Area of HV Winding is small
$\therefore R_{HV}$ is high

Ex (2)

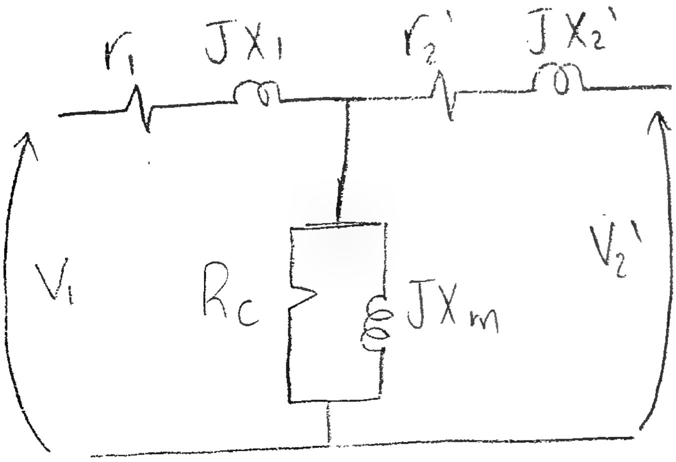
A 50 KVA, 2200/220 V transformer when tested give the following results:

Open circuit test: Measurments on LV side
405 W, 5 A, 220 V

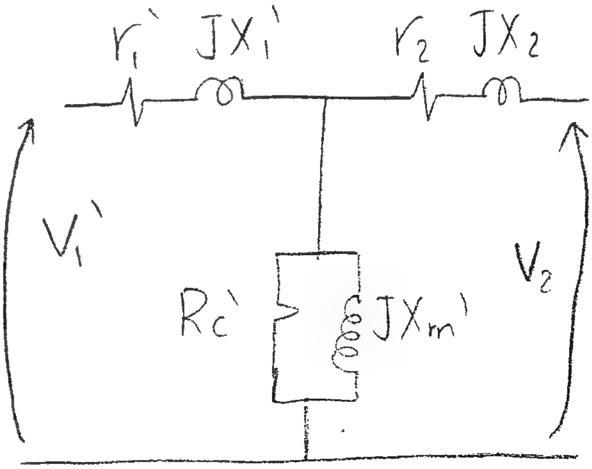
Short circuit test: Measurments on HV side
805 W, 20.2 A, 95 V

Find equivalent circuit parameters in ohms refered to HV side & LV side

Solution HV side → primary side in this problem



Exact equivalent circuit Referred to HV side



Exact equivalent circui Referred to LV side

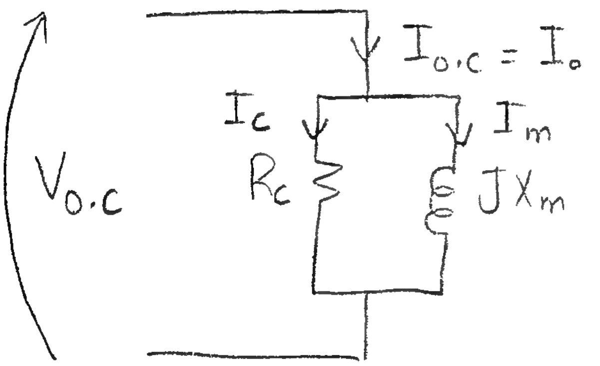
Solution

open Circuit test

$P_{o.c} = 405\,W$

$I_{o.c} = 5A$

$V_{o.c} = 220V$



$\rightarrow * \; P_{o.c} = V_{o.c} \, I_{o.c} \, \cos\phi$

$$\cos\phi = \frac{405}{220 * 5} = 0.368$$

$$\boxed{\phi = 68.4^\circ}$$

$\rightarrow I_c = I_{o.c} \cos\phi = 1.84$

$I_m = I_{o.c} \sin\phi = 4.65$

$$\rightarrow R_c\Big|_{LV} = \frac{V_{o.c}}{I_c} = 119.56\,\Omega$$

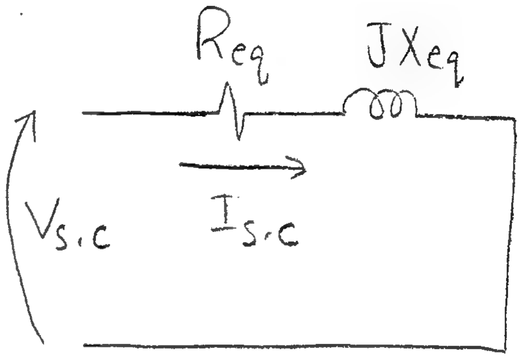
$$X_m\Big|_{LV} = \frac{V_{o.c}}{I_m} = 47.3\,\Omega$$

Short circuit test

$P_{s.c} = 805\,W$

$I_{s.c} = 20.2A$

$V_{s.c} = 95\,V$



$$\rightarrow R_{eq} = \frac{P}{I_{s.c}^2}$$

$$R_{eq}\Big|_{HV} = \frac{805}{(20.2)^2} = 1.97\,\Omega$$

$$\rightarrow Z_{eq} = \frac{V_{s.c}}{I_{s.c}} = 4.7$$

$$\rightarrow X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$

$$X_{eq}\Big|_{HV} = 4.26\,\Omega$$

$\therefore R_{eq}\Big|_{HV} = 1.97\,\Omega$

$X_{eq}\Big|_{HV} = 4.26\,\Omega$

Exact equivalent circuit referred to HV side

* $R_c|_{HV} = R_c|_{LV} \left(\frac{N_{HV}}{N_{LV}}\right)^2 = 119.56 \left(\frac{2200}{220}\right)^2 = 11.95\ K\Omega$

* $X_m|_{HV} = X_m|_{LV} \left(\frac{N_{HV}}{N_{LV}}\right)^2 = 47.3 \left(\frac{2200}{220}\right)^2 = 4.73\ K\Omega$

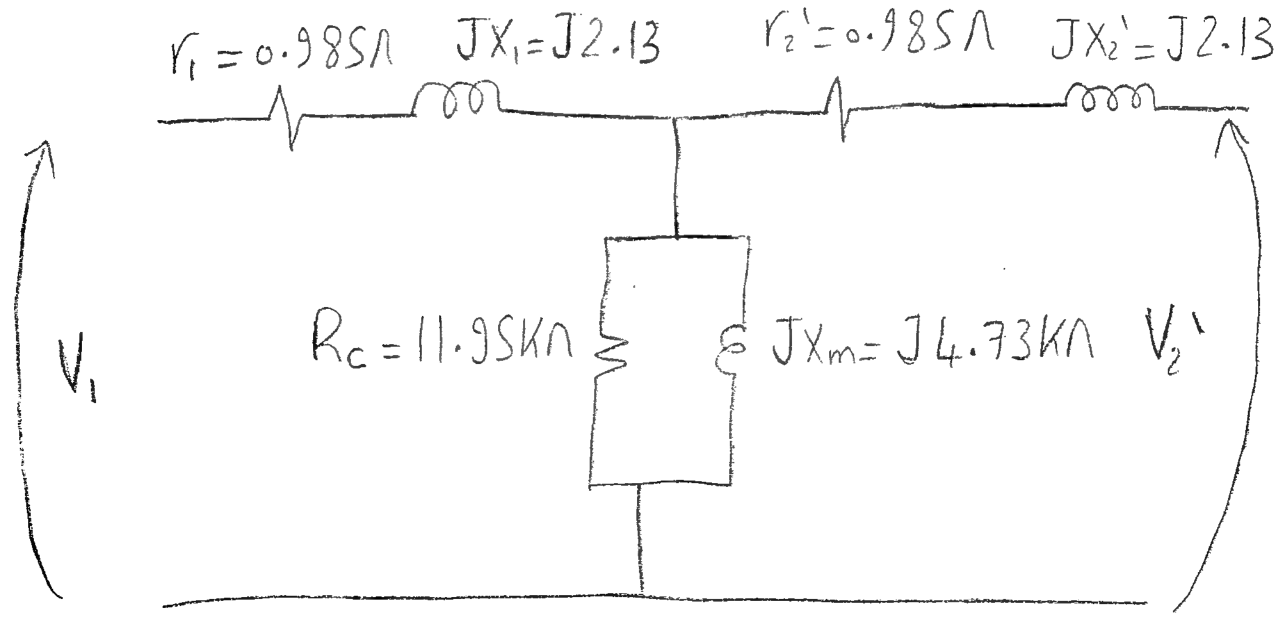
* $R_{eq}|_{HV} = 1.97$

$R_{eq}|_{HV} = r_1 + r_2' \quad (r_1 \simeq r_2')$

$\therefore r_1 = r_2' = \frac{R_{eq}|_{HV}}{2} = 0.985\ \Omega$

* $X_{eq}|_{HV} = 4.26$

$\therefore X_1 = X_2' = \frac{4.26}{2} = 2.13$

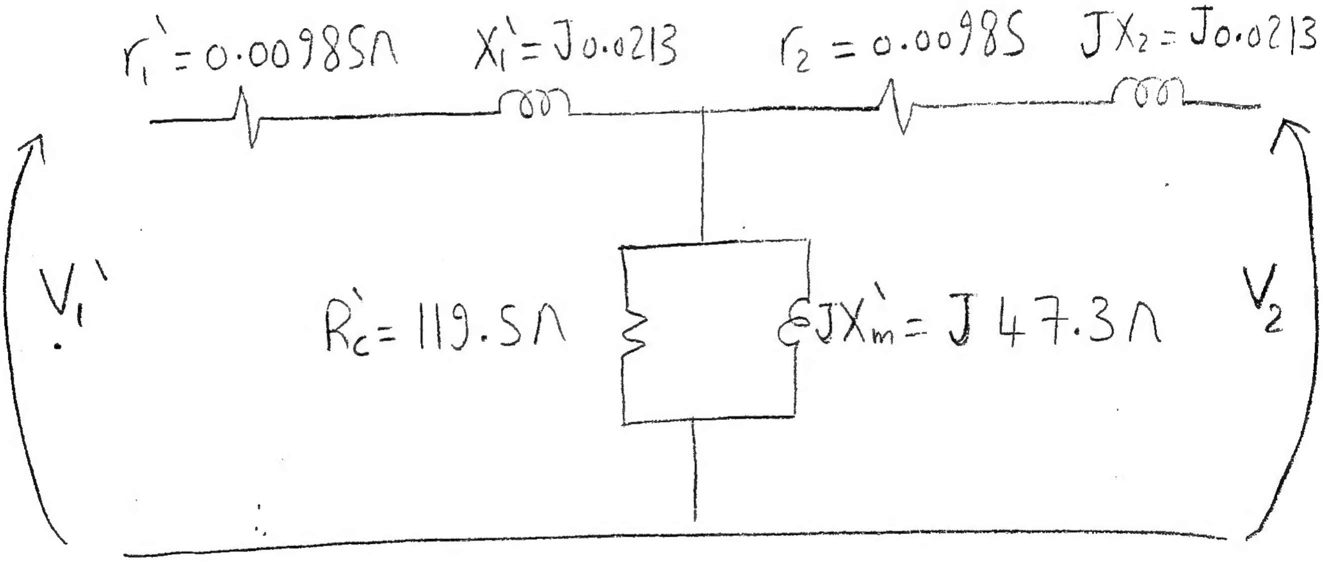


Exact equivalent circuit referred to LV side

Refering the circuit of HV side to LV side

$$r|_{LV} = r|_{HV}\left(\frac{N_{LV}}{N_{HV}}\right)^2 = r|_{HV}\left(\frac{220}{2200}\right)^2$$

$$r|_{LV} = \frac{r|_{HV}}{100}$$



Separation of iron losses

→ $P_{iron} = P_{core} = P_h + P_e$

→ where P_h: hystersis losses

P_e: eddy losses

* $P_h = K_h F (B_{max})^X$

X: Steinmetz Constant $(X=1.6)$

$$P_h = K_h F (B_{max})^{1.6}$$

حفظ

* $$P_e = K_e F^2 (B_{max})^2$$

حفظ

Where $B_{max} = \dfrac{V}{4.44 F N A}$

$\therefore P_h = K_h F \left(\dfrac{V}{4.44 F N A}\right)^X$

$P_h = K_h' V^X F^{1-X}$

$P_e = K_e F^2 \left(\dfrac{V}{4.44 F N A}\right)^2$

$P_e = K_e' V^2$

EX (3):

In a transformer, the Core loss is found to be 52 W at 40 HZ and 90W at 60 HZ measured at the same peak flux denisty. Compute the hystersis and eddy current losses at 50 HZ

Solution

$$P_h = K_h F (B_{max})^{1.6}$$

$\because B_{max} = \text{Constant}$

$$P_h = K_h' F$$

$$P_e = K_e F^2 B_{max}^2$$

$\because B_{max} = \text{Constant}$

$$\therefore P_e = K_e' F^2$$

$$\therefore P_{core} = P_h + P_e$$

$$P_{core} = K_h' F + K_e' F^2$$

using the given Conditions

$P_{core} = 52W$ at $40HZ \implies 52 = K_h'(40) + K_e'(40)^2 \to ($

$P_{core} = 90W$ at $60HZ \implies 90 = K_h'(60) + K_e'(60)^2 \to ($

Solving ①, ② $\Rightarrow$ $K_h' = 0.9$ $K_e' = 0.01$

$\therefore$ at 50 HZ

$$P_{core} = K_h'(50) + K_e'(50)^2$$

$$P_{core} = 0.9(50) + 0.01(50)^2$$

$$P_{core} = 45 + 25$$

$\therefore$ $P_{core} = 70W$

$P_h = 45W$

$P_e = 25W$